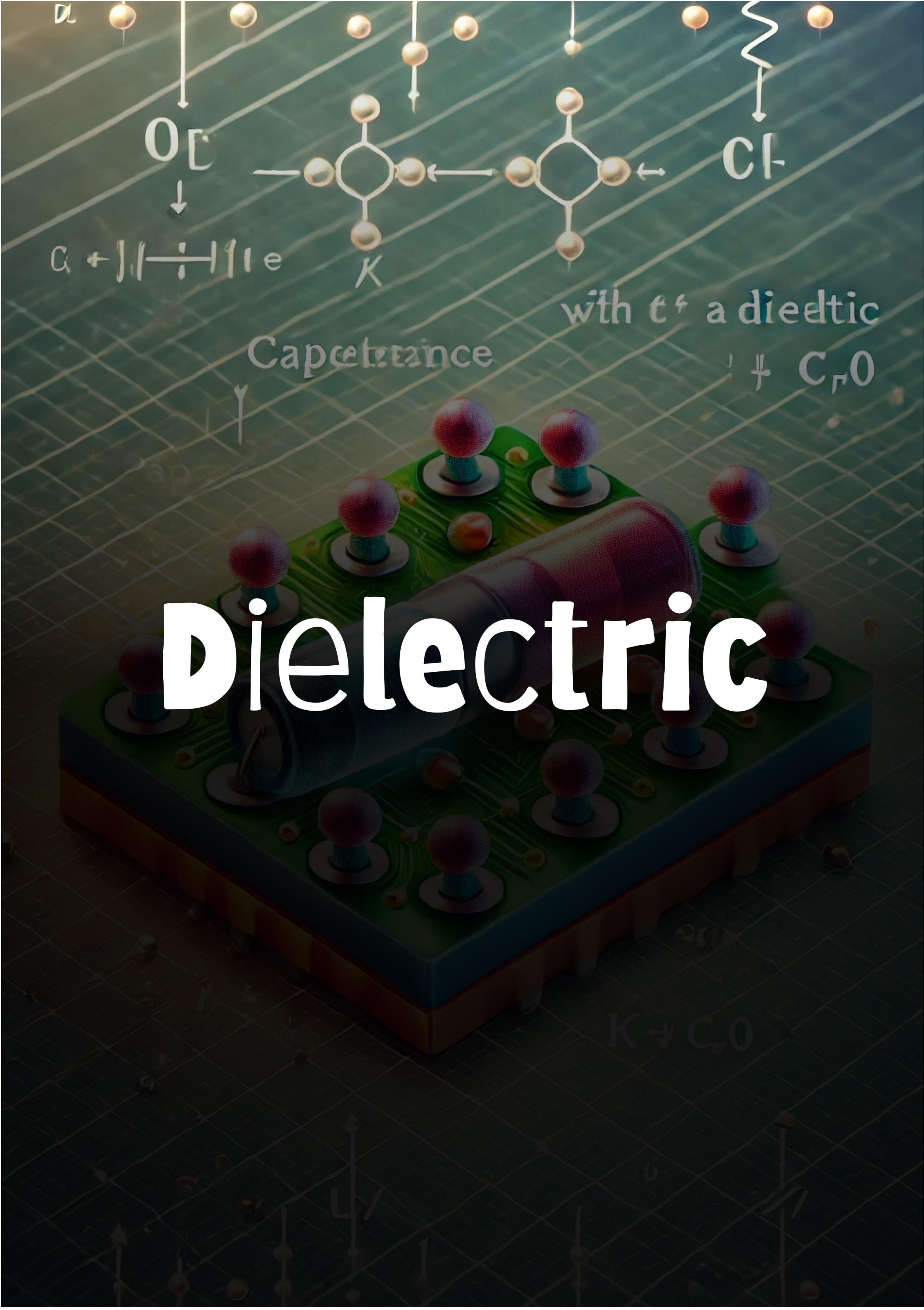
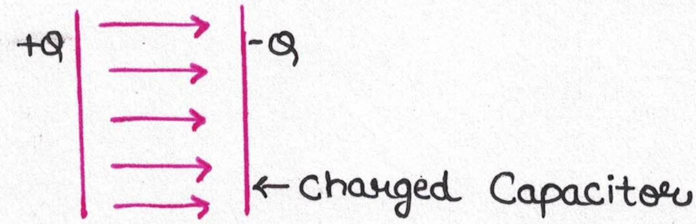


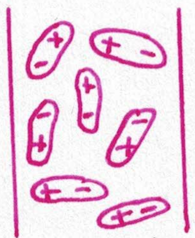
Dielectric

Dieelectrics

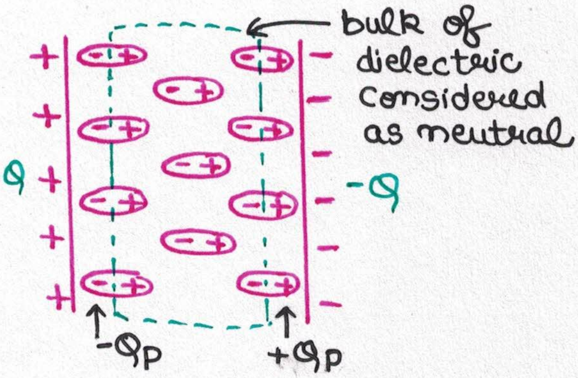
(1)



(2)



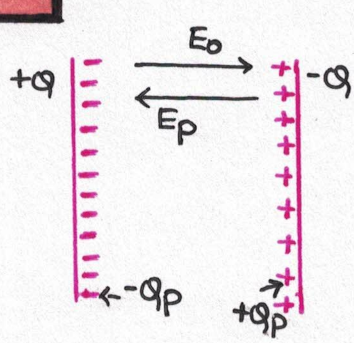
random but not zero



When we insert a polar substance (dielectric) b/w the plates of a charged capacitor then due to the field generated by the plates of the capacitor, the dipoles tend to align themselves such that their is an accumulation of net polarised -ve charge near the +ve plate & net polarised +ve charge near the -ve plate as shown in fig (3) (exaggerated)

It can be clearly seen from the fig. that except near the plates of the capacitor, bulk of the dielectric can be treated like neutral material.

EFFECT ON FIELD



$E_{net} = E_0 - E_p$

Let E_0 be the field in the capacitor, in the absence of dielectric.

Let E_p be the field due to the polarised charge in presence of dielectric. Then it can be clearly seen that net field in the region b/w the plates is given by

$$(E = E_0 - E_p)$$

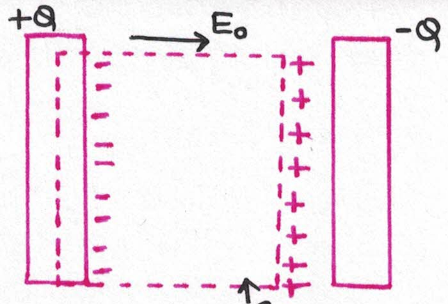
which is obviously less than E_0. Thus we see that whenever a dielectric is inserted b/w the capacitor plates, the field reduces by a certain factor. This factor is known as the dielectric constant (of the polar medium).

Mathematically, $K = \frac{E_0}{E}$ or $\boxed{E = \frac{E_0}{K}}$

It turns out that in the normal ranges of the field, the polarised field is directly proportional to the applied field & thus the ratio (E_0/E) almost remains constant. Hence the word 'Constant' and not variable.

However if the field becomes excessively large, there will be no longer a linear relationship b/w E_0 & E_P, & at very high fields the bonds in the dielectric may behave like a conductor. This phenomenon is called dielectric breakdown. For this reason, capacitors are provided with a voltage rating.

RELATION B/W PLATE CHARGE 'Q' & POLARISED CHARGE 'Qp'



$$\frac{Q - Q_P}{\epsilon_0} = \frac{E_0}{K}(A)$$

$$E_0 = \frac{Q}{A\epsilon_0}$$

$$Q - Q_P = \frac{Q}{K}$$

$$\boxed{Q_P = Q\left[1 - \frac{1}{K}\right]}$$

$$\boxed{\sigma_P = \sigma\left(1 - \frac{1}{K}\right)}$$

For metals,
$Q_P = Q$
$\because K = \infty$

For free space,
$K = 1$
$K_{air} \approx 1$

EFFECT OF A DIELECTRIC ON CAPACITANCE

$C = \frac{\epsilon_0 A}{d} = \frac{Q}{V}$ $\qquad$ $C' = \frac{Q}{V'}$

Since, $E' = \frac{E}{K}$ $\qquad$ $\therefore V' = \frac{V}{K}$

$C' = \frac{Q}{V/K} = K\left(\frac{Q}{V}\right)$

$$C' = KC = \frac{(K\epsilon_0)A}{d} = \frac{\epsilon A}{d}$$

NOTE: The product of ϵ_0 & K is called Permittivity of the dielectric.
Mathematically, $\epsilon = \epsilon_0 K$

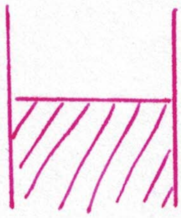
ENERGY DENSITY IN A DIELECTRIC

$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{\epsilon_0 K A}{d}(Ed)^2$

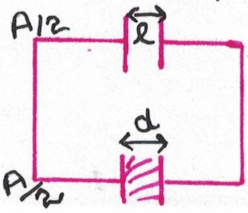
$u = \frac{U}{Ad} = \frac{1}{2}\frac{\epsilon_0 K A}{d}(Ed)^2 \cdot \frac{1}{Ad} = \frac{1}{2}\epsilon_0 K E^2$

$$u = \frac{1}{2}\epsilon E^2$$

Que.) A capacitor without the dielectric has a capacitance 'c'. What will be the new capacitance if half of the capacitor is filled with dielectric of constant 'K' as shown.



ΔV will be same as metal plates are equipotential. to maintain same field charge density in the given problem is equivalent to



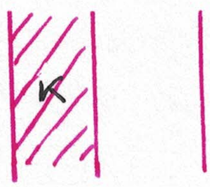
$$C' = \frac{\epsilon_0 (A/2)}{d} + \frac{\epsilon_0 K (A/2)}{d}$$

$$= \frac{\epsilon_0 A}{2d}(1+K)$$

Or $C' = C\left(\frac{1+K}{2}\right)$

$$\sigma_{lower} = K\, \sigma_{upper}$$

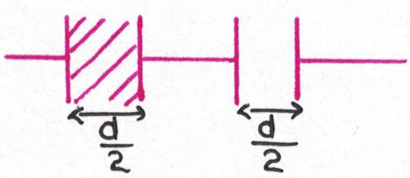
Que.) Repeat the last problem if half the capacitor was filled with dielectric as shown in fig.



applied charge is same
more field in right part

If we place a very thin conducting neutral sheet in the middle, C remains same as E, V & Q remain same. ie The sheet will not affect. Now trifurcate it.

Prob is equivalent to



$$C' = \frac{\epsilon_0 A K}{d/2}, \quad \frac{\epsilon_0 A}{d/2} \quad \text{in series}$$

$$= 2C \text{ in series with } 2KC$$

$$= \left(\frac{2\times 2K}{2+2K}\right)C \quad = \frac{2KC}{K+1}$$

Que.) What is Capacitance of system.

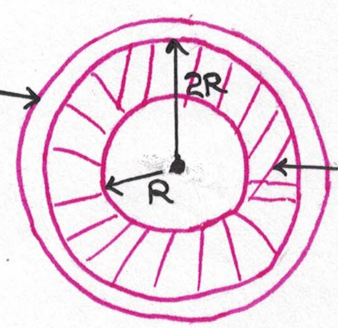
$$C_1 = \frac{4\pi\epsilon_0 \times 2R^2 K}{R}$$

$$C_1 = (8\pi\epsilon_0 R)K$$

$$C_2 = 4\pi\epsilon_0 (2R)$$

$$C_2 = 8\pi\epsilon_0 R$$

Thin sheet which does not affect Q & V can be assumed

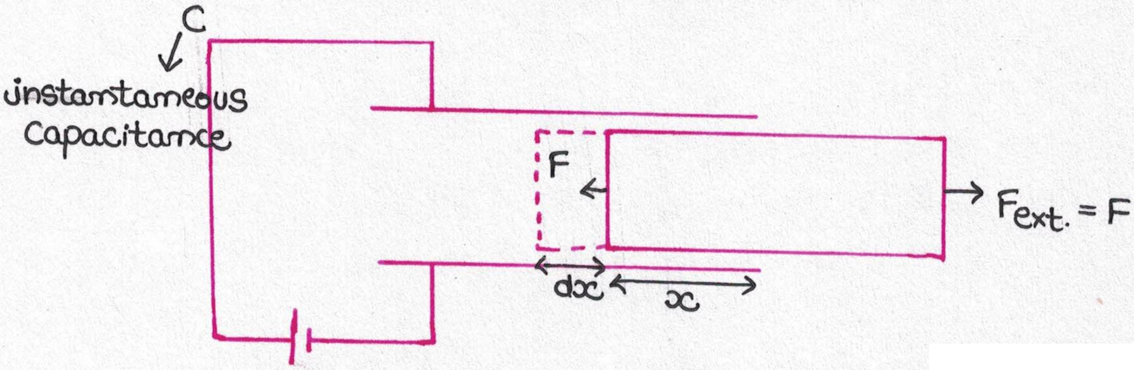


dielectric

$$C' = \frac{C \cdot K \cancel{q}}{(K+1)\cancel{q}} = \frac{K}{K+1} \cdot 8\pi\epsilon_0 R$$

$$C' = 8\pi\epsilon_0 R\left(\frac{K}{K+1}\right)$$

FORCE ON A DIELECTRIC



$$W_{ext} = -F\,dx$$

$$W_{battery} = v\,dq$$

$$U = \frac{1}{2}Cv^2$$

$$dU = \frac{1}{2}v^2 \cdot dC \qquad (\because V = \text{constant})$$

$$W_{ext} + W_{battery} \boxed{=} dU$$

↑ should be equal

$$-F\,dx + v\,dq = \frac{1}{2}v^2 dC$$

$$q = Cv$$

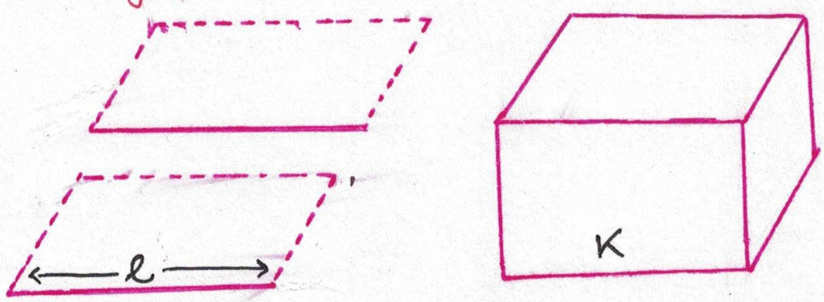
$$dq = v\,dC$$

$$-F\,dx + v^2 dC = \frac{1}{2}v^2 dC$$

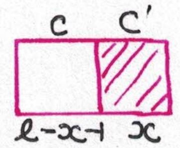
$$F\,dx = \frac{1}{2}v^2 dC$$

$$\boxed{F = \frac{1}{2}v^2 \frac{\partial C}{\partial x}}$$

Que.) A parallel plate capacitor has capacitance 'C' without the dielectric. Plate length is 'l'. Find the force exerted by this capacitor on a dielectric of constant 'K' having same dimensions as the space enclosed by capacitor. Battery voltage = V.



Let plate area = A



$$C' = \frac{\epsilon_0 (l-x)}{l}\frac{A}{d} + \frac{\epsilon_0 K x}{l}\frac{A}{d}$$

$$C' = \frac{\epsilon_0 A}{d l}\left[Kx + l - x\right]$$

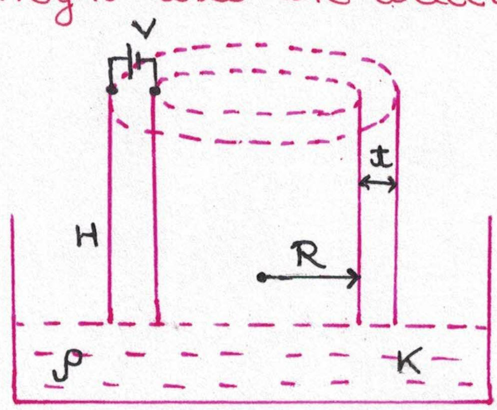
$$= \frac{C}{l}\left[x(K-1) + l\right]$$

$$F = \frac{1}{2} V^2 \frac{C}{l}(K-1)$$

$$F = \frac{1}{2}\cdot\frac{CV^2}{l}(K-1)$$

Oscillatory motion but not SHM, hence force will be constant.

Que.) To what height will the dielectric rise?



$t << R$

$$F = \frac{1}{2} V^2 \frac{\partial C}{\partial x}$$

$$W = 2\pi R h t \rho g$$

$$C' = \underset{\downarrow C_1}{\frac{x}{H}KC} + \underset{\downarrow C_2}{\left(\frac{H-x}{H}\right)C} \quad \text{in parallel}$$

$$C' = \frac{C}{H}\left[(K-1)x + H\right]$$

$$C' = \frac{2\pi\epsilon_0 H}{H \ln\left(\frac{R+t}{R}\right)}\left[(K-1)x+H\right]$$

$$\frac{\partial C}{\partial x} = \frac{2\pi\epsilon_0 R(K-1)}{t} \quad , \quad F = \frac{V^2\pi\epsilon_0 R(K-1)}{t}$$

For $x<<1$, $\ln(1+x) \approx x$

In case of (Q = Constant), we can write

$$F = \frac{1}{2}\frac{Q^2}{C^2}\cdot\frac{\partial C}{\partial x}$$

For the case of constant charge with a parallel rectangular plate capacitor, the force will depend on 'x' (amount which is inserted) unlike the case of constant voltage i.e. battery connected.